A five-point suspension from Mercedes

www.robotran.be

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1 Introduction

This exercise[1] consists in simulating the behaviour of a vehicle suspension. It aims to introduce, on a simplified model, all the necessary concepts for the modelling of a complete quad, namely:

- Multibody structure with rotation articulations.
- Link force and constitutive equations.
- Kinematic sensor.
- Ignorable variable.
- Fictitious body and forced-driven variable.

This problem has been proposed as an international vehicle benchmark in 1991 by the IAVSD 7. The complete description of the benchmark has been published in *Multibody Computer Codes in Vehicle System Dynamics*[2].

2 Data

2.1 Multibody model

- There are 8 bodies: 1 chassis, 5 rods, 1 carrier and 1 wheel (Fig. 1).
- The bodies are interconnected with each other by different "elementary" joints with 1 degree of freedom (prismatic or rotoid).
- The relative motion of the rods with respect to the base can take place in rotation about two different axis: longitudinal ($\hat{x}^0_x$) and vertical ($\hat{x}^0_z$) to the movement of the car.
- The relative motion of the carrier with respect of each rod can take place in 3 directions of rotation. To deal with system kinematic loops, *Ball* cuts (for more information, see RobotranTutorial) will be added between 4 rods and the carrier.
- The rotation movement of the wheel takes place around an axis which, in the reference configuration, is parallel to the transversal direction (y) of the suspension. Its associated coordinate does not take part in the equilibrium calculation, so it is ignored.

[1] Jean-Claude Samin and Paul Fisette : 2003, Symbolic Modeling Of Multibody Systems. Kluwer Academic Publishers.
[2] Kortüm, W. and R. Sharp: 1993, Multibody Computer Codes in Vehicle System Dynamics. Amsterdam: Swets & Zeitlinger.

- All these bodies are also subject to certain forces due to gravity, tire/ground contact, spring and suspension shock absorber. These last two suspension elements are modelled by a link force between the chassis and one suspension arm.

- The tire / ground contact is modelled by an external force acting on the wheel.

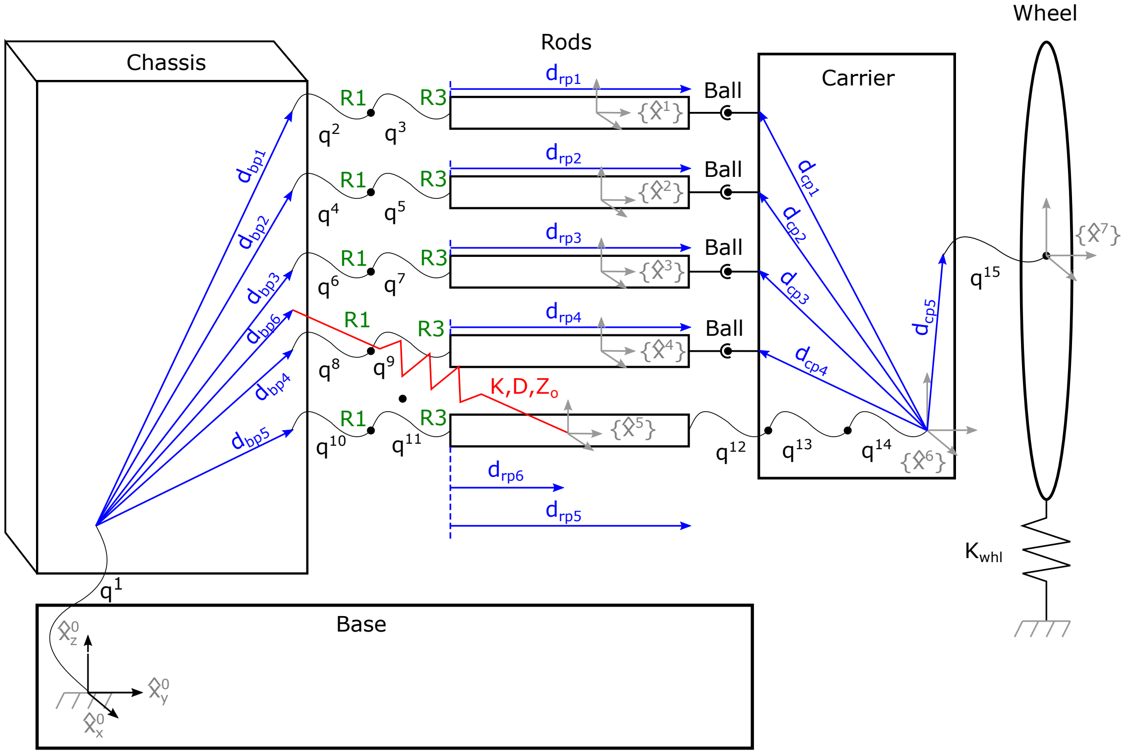

Figure 1: Multibody model

2.2 Contact modelling on flat ground

- The normal force behaviour law of the wheel/ground is:

$$F = max\{0, K_{whl} \cdot e\} \tag{1}$$

- The crushing e is calculated using the nominal radius r_{nom}, the height h of the wheel centre in the inertial coordinate system and the camber angle ϕ of the wheel.

$$e = r_{nom} \cdot cos(\phi) - h \tag{2}$$

- A kinematic sensor (F) in the centre of the wheel allows us to calculate:

  - The height of the centre of the wheel.
  - The camber angle, using the passage matrix R between the inertial and the wheel coordinate systems:

$$[\hat{x}^4] = [R][\hat{x}^0] \tag{3}$$

- The contact force is directed along the vertical of the inertial frame.
- Given the very low roll of the wheel, we assume that the line of action of the force passes through the centre of the wheel. This allows us to neglect the moment of the force with respect to the centre of mass of the wheel.

2.3 System data

- Base:
  - Gravity: $\mathrm{g} = [\hat{x}^0]^T \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$; $g = -9.81\ m/s^2$
- Chassis:
  - Mass : 409.91 Kg
  - Anchor point 1 position: $d_{bp1} = [\hat{x}^0]^T \begin{pmatrix} -0.064\ m \\ 0.413\ m \\ 0.327\ m \end{pmatrix}$
  - Anchor point 2 position: $d_{bp2} = [\hat{x}^0]^T \begin{pmatrix} -0.303\ m \\ 0.432\ m \\ 0.295\ m \end{pmatrix}$
  - Anchor point 3 position: $d_{bp3} = [\hat{x}^0]^T \begin{pmatrix} -0.093\ m \\ 0.366\ m \\ 0.004\ m \end{pmatrix}$
  - Anchor point 4 position: $d_{bp4} = [\hat{x}^0]^T \begin{pmatrix} -0.236\ m \\ 0.388\ m \\ -0.109\ m \end{pmatrix}$
  - Anchor point 5 position: $d_{bp5} = [\hat{x}^0]^T \begin{pmatrix} -0.2115\ m \\ 0.3845\ m \\ -0.100\ m \end{pmatrix}$
  - Anchor point 6 position (where the link is attached to): $d_{bp6} = [\hat{x}^0]^T \begin{pmatrix} 0.090\ m \\ 0.525\ m \\ 0.170\ m \end{pmatrix}$
- Joints Chassis-Rods:
  - Joint type: R1 & R3
- Rods bodies:
  - Rod1 mass: 2 kg
  - Rod2 mass: 2 kg
  - Rod3 mass: 4 kg
  - Rod4 mass: 1 kg
  - Rod5 mass: 3 kg

- Centre of mass position of each one:

$$\Big|\mathrm{COM}\Big|_1 = [\hat{x}^1]^T \begin{pmatrix} 0\ m \\ 0.1118626389819228\ m \\ 0\ m \end{pmatrix}$$

$$\Big|\mathrm{COM}\Big|_2 = [\hat{x}^2]^T \begin{pmatrix} 0\ m \\ 0.1396969935252724\ m \\ 0\ m \end{pmatrix}$$

$$\Big|\mathrm{COM}\Big|_3 = [\hat{x}^3]^T \begin{pmatrix} 0\ m \\ 0.1489253168537841\ m \\ 0\ m \end{pmatrix}$$

$$\Big|\mathrm{COM}\Big|_4 = [\hat{x}^4]^T \begin{pmatrix} 0\ m \\ 0.2095250581672751\ m \\ 0\ m \end{pmatrix}$$

$$\Big|\mathrm{COM}\Big|_5 = [\hat{x}^5]^T \begin{pmatrix} 0\ m \\ 0.2056047482428361\ m \\ 0\ m \end{pmatrix}$$

- The inertia effects of the bars are neglected.
- Anchor point 1 (where the Ball 1 is attached to): $d_{rp1} = [\hat{x}^1]^T \begin{pmatrix} 0\ m \\ 0.2237\ m \\ 0\ m \end{pmatrix}$
- Anchor point 2 (where the Ball 2 is attached to): $d_{rp2} = [\hat{x}^2]^T \begin{pmatrix} 0\ m \\ 0.2794\ m \\ 0\ m \end{pmatrix}$
- Anchor point 3 (where the Ball 3 is attached to): $d_{rp3} = [\hat{x}^3]^T \begin{pmatrix} 0\ m \\ 0.2979\ m \\ 0\ m \end{pmatrix}$
- Anchor point 4 (where the Ball 4 is attached to): $d_{rp4} = [\hat{x}^4]^T \begin{pmatrix} 0\ m \\ 0.4191\ m \\ 0\ m \end{pmatrix}$
- Anchor point 5 (where the wheel joint is attached to): $d_{rp5} = [\hat{x}^5]^T \begin{pmatrix} 0\ m \\ 0.4116\ m \\ 0\ m \end{pmatrix}$

• Joint between the Rod 5 and the Carrier:
  - Joint type: R1 & R2 & R3

• Cuts between the Rods 1-4 and the Carrier:
  - Ball cut 1 between the Rod1 and the Carrier
  - Ball cut 2 between the Rod2 and the Carrier
  - Ball cut 3 between the Rod3 and the Carrier
  - Ball cut 4 between the Rod4 and the Carrier
  - Ball cut 5 between the Rod5 and the Carrier

• Carrier body:

- Mass: 15 kg
- Centre of mass: $\Big|\mathrm{COM}\Big|_6 = [\hat{x}^6]^T \begin{pmatrix} -0.1055\ m \\ -0.2713\ m \\ 0.2243\ m \end{pmatrix}$
- Inertia: $I^6_{xx} = 0.3\ kg \cdot m^2$; $I^6_{yy} = 0.20122\ kg \cdot m^2$; $I^6_{zz} = 0.198780669\ kg \cdot m^2$; $I^6_{xy} = I^6_{yx} = 0.0101821697\ kg \cdot m^2$; $I^6_{yz} = I^6_{zy} = 0.0499851301\ kg \cdot m^2$; $I^6_{xz} = I^6_{zx} = 0.0148769426$
- Anchor point 1 (where the Ball 1 is attached to): $d_{cp1} = [\hat{x}^6]^T \begin{pmatrix} -0.0665\ m \\ -0.0962051898607574\ m \\ 0.4799860012998876\ m \end{pmatrix}$
- Anchor point 2 (where the Ball 2 is attached to): $d_{cp2} = [\hat{x}^6]^T \begin{pmatrix} -0.1195\ m \\ -0.0962751863610199\ m \\ 0.4729863512736397\ m \end{pmatrix}$
- Anchor point 3 (where the Ball 3 is attached to): $d_{cp3} = [\hat{x}^6]^T \begin{pmatrix} -0.1905\ m \\ -0.0888855558333097\ m \\ 0.11189440541959\ m \end{pmatrix}$
- Anchor point 4 (where the Ball 4 is attached to): $d_{cp4} = [\hat{x}^6]^T \begin{pmatrix} -0.0075\ m \\ 3.999800015 \cdot 10^{-5}\ m \\ 3.9998000149988 \cdot 10^{-3}\ m \end{pmatrix}$
- Anchor point 5 (where the wheel joint is attached to): $d_{cp5} = [\hat{x}^6]^T \begin{pmatrix} -0.0025\ m \\ 0.0323383831212649\ m \\ 0.1336833160012957\ m \end{pmatrix}$

- Wheel rotation joint:
  - Joint type: R2
- Wheel body:
  - Mass: 12 kg
  - Inertia: $I^7_{xx} = 0.74\ kg \cdot m^2$; $I^7_{yy} = 1.46\ kg \cdot m^2$; $I^7_{zz} = 0.74\ kg \cdot m^2$
- Suspention:
  - Stiffness: $K = 1.4 \cdot 10^5\ N/m$
  - Damping: $D = 1400\ N \cdot s/m$
  - Neutral length: $Z_o = 0.892371964\ m$
- Tyre/ground contact:
  - Stiffness: $K = 2 \cdot 10^5\ N/m$
  - Nominal radius: $r_{nom} = 0.30\ m$

2.4 Initial conditions

As the suspension has one degree of freedom, there is only one independent joint variable. The chosen variable is q^4, which is the rotation around the longitudinal axis (*R1*) between the chassis ("Base") and the second rod ("Rod2").

For this forced configuration, a basic geometrical analysis gives the next value for the independent variable: $q^{11} = 0.207743260032\ rad$. For the dependent variables, the proposed configuration gives the following initial guess values:

Variable	Valeur initiale
q^1	$0.278\ m$
q^2	$4.613882548811577\ (\times\ \pi/180)$
q^3	$4.02889045738623e-05\ (\times\ \pi/180)$
q^4	$11.902812022122399\ (\times\ \pi/180)$
q^5	$-41.73804367345049\ (\times\ \pi/180)$
q^6	$-5.489179473987941\ (\times\ \pi/180)$
q^7	$18.599571106148783\ (\times\ \pi/180)$
q^8	$3.444025498268294\ (\times\ \pi/180)$
q^9	$-33.452591013228314\ (\times\ \pi/180)$
q^{10}	$-5.509358437021373\ (\times\ \pi/180)$
q^{11}	$30.547781981310944\ (\times\ \pi/180)$
q^{12}	$5.347518201452412\ (\times\ \pi/180)$
q^{13}	$-3.143760886564701\ (\times\ \pi/180)$
q^{14}	$-30.365880376422055\ (\times\ \pi/180)$

Table 1: Initial values of dependent variables.

2.5 Simulations definition

- Kinematics stage: two different analysis are implemented.
  - First of all, a MbsSolvekin module is used to define the first kinematics to reach the lower bumpstop. For this step, the independent variable of the rotation of the second rod is converted to a driven variable and the control law is the fallawing :

    $$q = (q_{min} - q_0)\, t_{sim} + q_0$$

    with $q_{min} = -1.1$ rad and $q_0 = 0.207743260032$ rad.
  - Secondly, a MbsDirdyn module (direct dynamics) is used to study the evolution of the suspension when it goes from the lower to upper bumpstop, using the following control law :

    $$q = (q_{max} - q_{min})\, tsim + q_{min}$$

    where $q_{max} = 1.2$ rad and $q_{min} = -1.1$ rad
- First dynamics analysis: the ground undergoes a downwards step of 10 cm after 0,2 seconds of simulation. The rotation variable along the longitudinal axis of the second rod is set to independent,
- Second dynamics analysis: a new body, representing a vibrating poster, is added to the model.
  - This body is located in the same lateral position as the wheel (0,78 m).
  - It is connected to the base with a vertical coordinate (T3) defined as a Forced-Driven variable.
  - The vertical vibration of the poster is defined by the following *sweep* function.

    $$z(t) = z_{max}\ sin\left(2\pi \left(f_0 + \frac{f_1 - f_0}{t_1 - t_0}\frac{t}{2}\right) t\right)$$

    avec $z_{max} = 5\ mm$, $f_0 = 1\ Hz$ en $t_0 = 0\ s$ et $f_1 = 10\ Hz$ en $t_1 = 10\ s$.

  - The vertical translation of the chassis is set to independent variable.

3 Objectives

3.1 Kinematics

In this section, the objective is to analyse the evolution of the camber angle (Fig. 2), the Toe angle (Fig. 3) the longitudinal position (Fig. 4) and the lateral position (Fig. 5) when the height of the wheel centre is altered within a range of plus or minus 20 cm around the equilibrium configuration[3]:

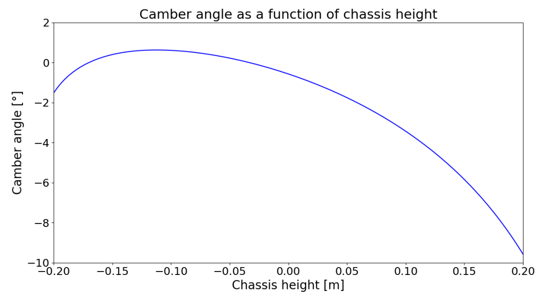

Figure 2: Camber angle

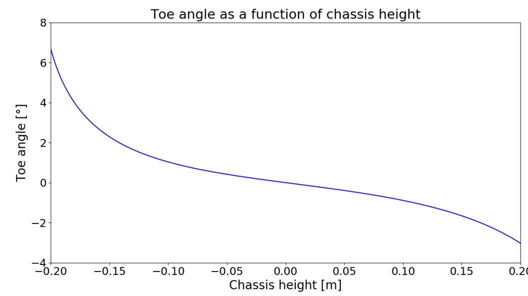

Figure 3: Toe angle

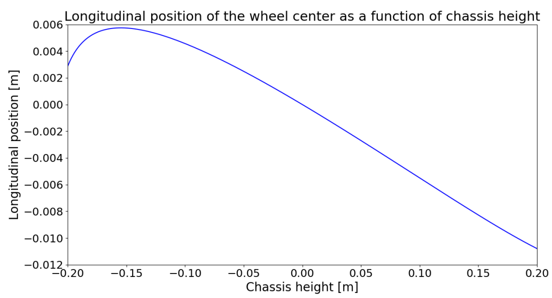

Figure 4: Evolution of the longitudinal position (x) of the wheel centre

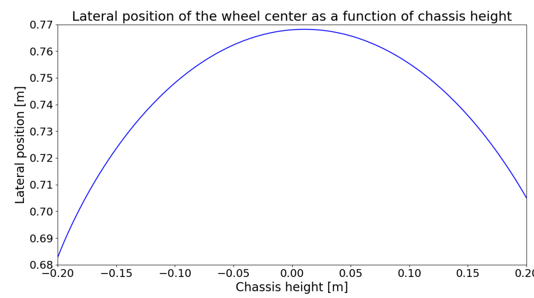

Figure 5: Evolution of the lateral position (y) of the wheel centre

[3]Jean-Claude Samin and Paul Fisette : 2003, Symbolic Modeling Of Multibody Systems. Kluwer Academic Publishers.

3.2 Dynamics I

In the equilibrium position, the unloaded length of the suspension's spring is 0.8918286961474595 m. The aim here is to find the time evolution of the vertical displacement (Fig. 6), velocity (Fig. 7) and acceleration (Fig. 8) of the wheel centre when the ground undergoes a downwards step of 10 cm.

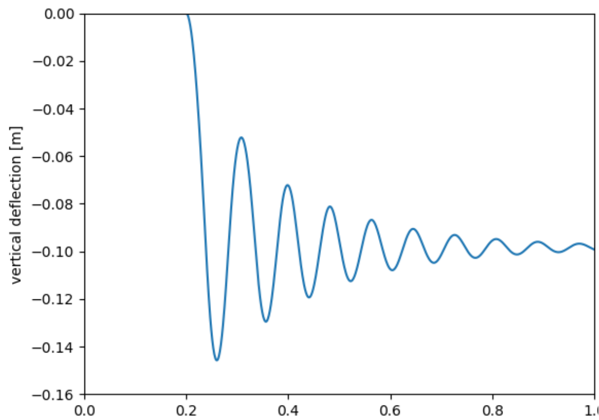

Figure 6: Vertical (z) position of the wheel centre

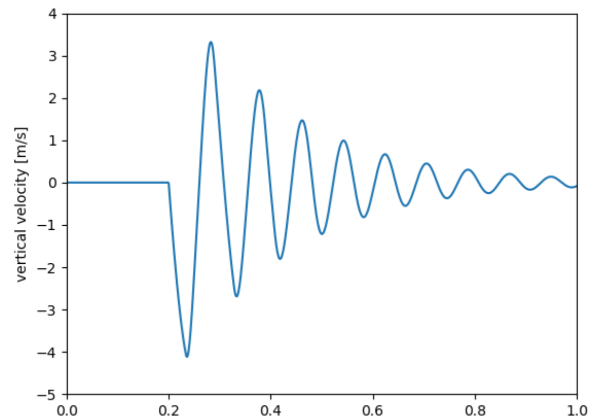

Figure 7: Vertical (z) velocity of the wheel centre

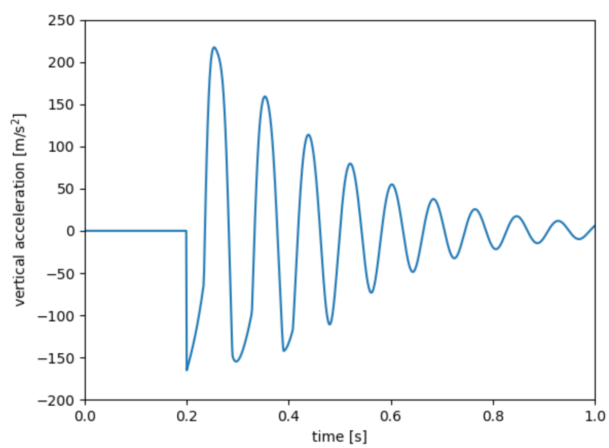

Figure 8: Vertical (z) acceleration of the wheel centre

3.3 Dynamics II

A vertical vibration is introduced to the wheel by the poster. For this configuration, the time evolution of the vertical displacement, velocity and acceleration (Fig. 9) of the wheel centre are represented for the first 20 seconds.

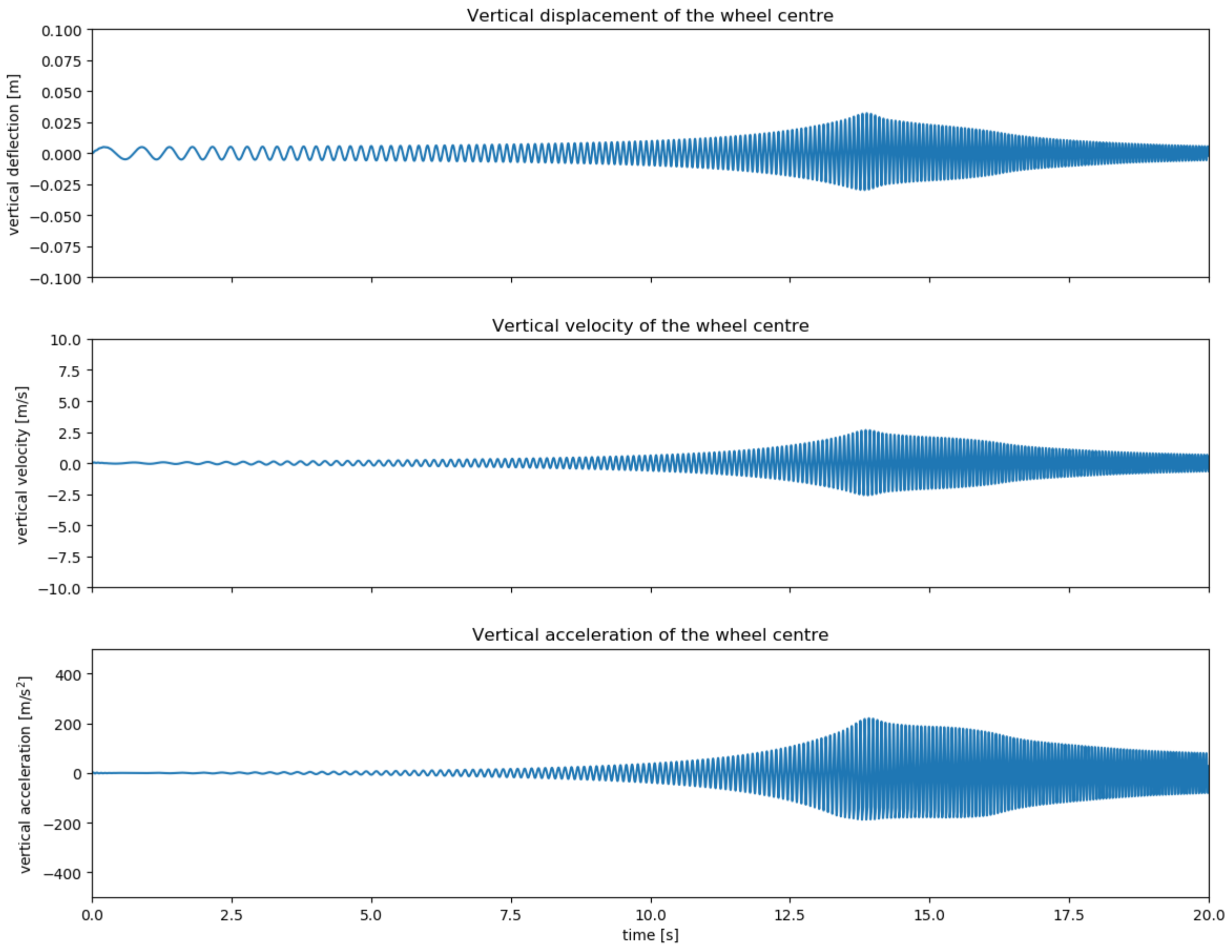

Figure 9: Vertical position of the wheel centre and its derivatives